An Attempt at a Theory of "Beta" Ray Emission[1]

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SUMMARY: A theory of the emission of β-rays from radioactive substances, founded on the hypothesis that the electron emitted by the nucleus does not exist prior to the decay, but rather is created by it, along with a neutrino[2], in a manner analogous to the creation of a light-quantum that accompanies the quantum jump of an atom. Comparison of theory to experiment.

I propose to show here the foundation of a theory of β-ray emission that—although based on a hypothesis that lacks, at the present moment, any experimental confirmation—nevertheless appears to be capable of giving a sufficiently accurate representation of the facts, and permits a quantitative treatment of the behavior of the nuclear electron which, even if the fundamental hypothesis of this theory should prove to be false, may yet in any case serve as a useful guide for directing experimental research.

It is well known that in seeking to construct a theory of β-rays, one encounters at first a difficulty depending on the fact that the β-ray escapes from the radioactive nucleus with a continuous velocity distribution whose limit extends to a certain maximum velocity: this at first glance does not appear to be compatible with the principle of conservation of energy. A qualitative possibility to explain this fact without being forced to abandon the principle of conservation of energy consists, according to Pauli, in admitting the existence of something like the aforementioned "neutrino", that is to say an electrically neutral particle, of a mass on the order of magnitude of the electron or less. In each β decay, we have the simultaneous emission of an electron and a neutrino; and the energy liberated in this process is divided between the two particles in such a way that the energy of the electron can take on all of the values from zero up to a certain maximum. The neutrino, on the other hand, because of its electrical neutrality and its tiny mass, has such a high penetrating power that it escapes almost every actual method of observation. In the theory that we propose, we adopt the point of view of the hypothesis of the existence of the neutrino.

Apart from the difficulty of the continuous distribution of energy, a theory of β-rays encounters another essential difficulty in the fact that the present theory of light particles does not explain, in a satisfactory manner, how they can be bound in a stable or quasi-stable manner inside the nucleus, given its small volume.

The simplest way to construct a theory that permits a quantitative discussion of the phenomena in which the nuclear electron takes part appears, consequently, to be to pursue the hypothesis that the electron does not exist as such in the nucleus before β-emission, but rather, as so stated, that it acquires existence at the precise moment in

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[1] Translator's Note: This may be the first paper Fermi published concerning the neutrino. A few months later, he published two longer, quantitatively identical versions of this paper—*Nuovo Cimento* **11**, 1 (1934) and *Zeitschrift f. Physik* **88,** 161 (1934). In these notes, the latter is referred to as the "*Zeitschrift"* paper.

[2] Tr. Note: In modern language, this is an anti-neutrino.

which it is emitted—in exactly the same way in which a light-quantum emitted from an atom in a quantum jump is not in any way considered to have pre-existed in the atom prior to the emission process. In this theory, therefore, the total number of electrons and neutrons (just like the total number of light-quanta in the theory of radiation) will not necessarily be constant, but will have the potential for processes of creation or destruction of light particles.

Following Heisenberg's idea, we consider the heavy particles, the neutron and the proton, to be like two quantum states associated with two possible values of an internal coordinate ρ of a heavy particle. To it, we attribute the value +1 if the particle is a neutron, and –1 if the particle is a proton.

We seek an expression for the energy of interaction between the light particles and the heavy ones that agrees with the transition between the two values +1 and –1 of the coordinate ρ, which is the transformation from a neutron to a proton or vice-versa; yet in such a way that the transformation from a neutron to a proton is of necessity tied to the creation of an electron, which is observed as a β-particle, and to a neutrino; while the inverse transformation of a proton to a neutron is connected to the destruction of an electron and a neutrino; just as in the theory of radiation a certain quantum jump of the atom is connected to the emission of a light-quantum, while the opposite quantum jump is associated with the absorption of the quantum.

The simplest mathematical formalism for constructing a theory in which the number of particles (electron and neutrino) is not necessarily constant, is found in the method of Dirac-Jordan-Klein of the "amplitude of quantized probability." In this formalism, the probability amplitude ψ of the electron and φ of the neutrino[3], and their complex conjugates ψ^* and φ^*, are considered as non-commutative operators that act on the functions of the occupation numbers of the quantum states of the electron and the neutrino; in this way, the operator ψ determines transitions in which the number of electrons decreases by one unit, while its complex conjugate operator ψ^* determines the inverse transition in which the total number of electrons is increased by one. In the usual application of this method, naturally, the operators ψ and ψ^* are always associated one with the other, for in this way, in the processes that one considers, the total number of particles remains constant. Instead, in the present theory, the possibility of a change in the number of particles one obtains introduces the two inverse operators in separate terms in the interaction energy.

We still must introduce two other operators Q and Q^* that operate on the functions of the variable with two values ρ like the linear substitutions:

$$Q = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \qquad Q^* = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}$$

[3] Tr. Note: In modern notation, the probability amplitude of the anti-neutrino is the *adjoint* operator $\overline{\varphi}$.

These operators, as is easily recognized, determine respectively the transition from a proton to a neutron and from a neutron to a proton[4].

An expression for the interaction energy that necessarily associates the transition of a neutron to a proton (operator *Q**) with the creation of an electron and a neutrino, and necessarily associates the transition from a proton to a neutron (operator *Q*) with the disappearance of an electron and a neutrino (operators ψ and φ) is in its most general form the following:

$$H = QL(\psi,\varphi) + Q^* L^*(\psi^*,\varphi^*)$$

In which *L* represents an expression bilinear in ψ and φ, which could eventually contain also the coordinates, the momenta, and the spin coordinates of the heavy particles. One obtains a constraint on the possible choices of *L* when looking for those expressions that, under a change of frame of reference, transform as the time component of a four-vector. One can demonstrate that the simplest of such expressions[5] is the following:

$$L(\psi,\varphi) = g(\psi_2\varphi_1 - \psi_1\varphi_2 + \psi_3\varphi_4 - \psi_4\varphi_3)$$

where the ψ and the φ are the four components of the relativistic probability amplitude[6] of the electron and neutrino, and *g* represents a constant of proportionality. Since the outcome of this choice is shown to be in good agreement with the experimental facts, one need not at the moment look into more complicated expressions.

Thus, when all is said and done, we take the expression for the interaction the following.

$$(1) \qquad H = g\left\{Q\left(\psi_2\varphi_1 - \psi_1\varphi_2 + \psi_3\varphi_4 - \psi_4\varphi_3\right) + Q^*\left(\psi_2^*\varphi_1^* - \psi_1^*\varphi_2^* + \psi_3^*\varphi_4^* - \psi_4^*\varphi_3^*\right)\right\}$$

In this expression, ψ and φ (considered as operators) should be evaluated at the point in space occupied by the heavy particle. The constant *g* that appears in (1) has dimensions of L^5MT^{-2}.

Taking (1) as the expression for the interaction energy, one can construct a theory of β-decay, by a method similar to that used in the theory of radiation, to calculate the half-life of an excited state of an atom. Without here entering into the mathematical details of the theory, we will confine ourselves only to observe that the de Broglie wavelength, for light particles having energies no greater than a few million volts, is large compared to nuclear dimensions. It follows that, to a first approximation, we can neglect the variation of ψ and φ between different points of the nucleus; this corresponds in the theory of radiation to neglecting quadrupole radiation. Making this approximation, it is

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[4] Tr. Note: Fermi takes these operators from Heisenberg, who derived them from the Pauli matrices: in modern notation we call them $\sigma_\pm$. In modern notation, the "internal coordinate" ρ is discarded and the operators *Q* and *Q** are replaced with creation and destruction operators for protons and neutrons.

[5] Tr. Note: In the *Zeitschrift* paper Fermi discusses the sixteen possible bilinear combinations of these components as candidates.

[6] Tr. Note: These components arise from Dirac's relativistic electron theory, which Fermi had recently summarized in *Rev. Mod. Phys*., **4**, 87-132 (1932).

found that the half-life for a β-decay process—in which a neutron, bound in an orbit with eigenfunction u_n, is transformed into a proton belonging to a quantum state v_m, emitting a β-ray and a neutrino—is given by:

$$(2) \qquad \frac{1}{\tau} = 1.65 \times 10^{95} g^2 \left(10^{12} r\right)^{-0.4} \left| \int u_n v_m^* d\tau \right|^2 F(\eta_0)$$

In this expression, $mc\eta_0$ represents the maximum momentum of the emitted electron, which is determined by the difference in energy between the two states u_n and v_m of the heavy particle, and r is a wavelength on the order of 10^{-12} cm, whose exact value depends on the assumption one makes about the nature of the electric potential inside the nucleus[7]. The function F has a rather complicated analytic expression; for small values of the argument it behaves almost exactly like $\eta_0^6/24$, while for bigger arguments, it takes on the values that are given in the following table:

η_0	$F(\eta_0)$
1	*0.03*
2	*1.2*
3	*7*
4	*29*
5	*80*
6	*185*
7	*380*

These values, like also equation (2) , are states calculated for the atomic number Z = 82; yet they do not vary considerably among the small interval of atomic numbers represented in the radioactive family. Moreover, in (2) the mass of the neutrino is set precisely equal to zero. Since, to yield an agreement between the theoretical and experimental curves[8] that give the continuous distribution of energy of the β-particle, one finds that, in order to have this agreement, it is necessary to allow the mass of the neutrino to be much less than that of the electron—the simplest hypothesis consists in supposing that it really is equal to zero.

The only unknown element in (2) is the matrix element

$$q = \int u_n v_m^* d\tau$$

the calculation of which requires that we know the eigenfunctions u_n and v_m belonging to the two states of the neutron and the proton inside the nucleus. Nevertheless, one can assert that q is of the order of unity; only in cases of particular symmetry of the two wavefunctions u_n and v_m can q be exactly zero. This case corresponds to forbidden

[7] Tr. Note: From the *Zeitschrift* paper, it is clear that the factor of $(10^{12} r)^{0.4}$ arises from an approximate treatment of Coulomb effects on the electronic wavefunction.

[8] Tr. Note: In Figure 1, Section 7 of the *Zeitschrift* paper, Fermi illustrates the dependence of the energy distribution of the electron on the hypothetical neutrino mass.

optical transitions, in which one cancels the element corresponding to the transition considered in the matrix that represents the electric moment. In that case, the approximation that we made in (2) is not sufficient, and one must also take into consideration the terms that depend on the variation of ψ and φ over the range of the nucleus. A calculation of this kind, completely analogous to the consideration of quadrupole terms in the theory of radiation, leads one to predict that in this case the intensity of the β-transition is, to an order of magnitude, several hundreds of volts smaller than in the usual case.

We must therefore expect that if, for several bodies which decay by emitting β-rays, one forms the product $\tau F(\eta_0)$, that they will be all of the same order of magnitude; except for the case in which the β-transition, in the same sense of which we have been speaking, cannot happen in the first approximation. In this case, the product $\tau F(\eta_0)$ should take on values that are several hundreds of volts larger than normal.

In the following table[9] are gathered the products $\tau F(\eta_0)$ for all of the substances that emit β-rays for which one has sufficient data; τ is expressed in hours.

Element		τ ***F***(η_0)	**Element**		τ ***F***(η_0)
Ra B	[Pb^{214}]	0.62 {0.9}	Ra C	[Bi^{214}]	130 {190}
Th B	[Pb^{212}]	1.7 {2.7}	Ra E	[Bi^{210}]	770 {1800}
Th C″	[Tl^{208}]	2.9 {3.3}	Ms Th_2	[Ac^{228}]	180 {640}
Ac B	[Bi^{211}]	0.7 {0.09}	Th C	[Bi^{212}]	150 {230}
Ac C″	[Tl^{207}]	1.4 {2.0}			
UX_1	[Th^{234}]	3.3 {5.4}			
UX_2	[Pa^{234}]	3.3 {3.0}			

In this table, the two groups that we expected are clearly recognizable. One also notes that the elements that occupy corresponding positions in different radioactive families always belong to the same group.

By these results, it is also possible to calculate at least the order of magnitude of the constant g.[10] The result is 5×10^{-50}.

Finally, the theory permits one to calculate the shape of the distribution of velocities of the β-particles. Since the experimental data, particularly as regards the low-energy part of the curve, are sometimes contradictory, it is not possible to base on this a precise check on the theory; yet it still shows good qualitative agreement. In particular, the theory implies that the distribution curve for small velocities should go to zero faster for those elements in which the transition is prohibited to a first approximation—that is to say, for those members of the second group of the proceeding table, not for the others. The experimental data does not seem to be sufficiently definitive for a test of this point.

A more extensive exposition of this theory and its further results will be published next in another place.

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[9] Tr. Note: I have listed the modern names of these elements in [*brackets*] next to their original names. In the *Zeitschrift* paper, Fermi uses data from B. W. Sargent, *Proc. Royal. Soc* **139,** 659 (1933); I have listed these revised values in {braces} next to their original values, whose source is unknown.

[10] Tr. Note: In the *Zeitschrift* paper, the order of magnitude estimate is 4×10^{-50}.